Mathematics Applied to Quantitative Social Sciences Intermediate Level

Session 4

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Contents

Examples

Tangent lines

A.1 First compute the derivative of $f(x) = x^3$ using limits and then use it to find the slope of the tangent line to the curve $y = x^3$ at the point where $x = -1$. What is the equation of the tangent line at this point?

A.2 Find an equation for the tangent line to the graph of $f(x) = x - \ln \sqrt{x}$ at the point where $x = 1$.

A.3 For the function $y = (2x + 1)(2x^2 - x - 1)$:

- A.3.1 Find y'
- A.3.2 Find an equation for the tangent line to the curve at the point where $x = 1$
- A.3.3 Find all points on the curve where the tangent line is horizontal.

Examples

Derivatives and Differentiability

B.1 Consider a firm with a production function $y = \frac{1}{2}\sqrt{x}$. Suppose it cuts its labor force from 900 to 896 units. estimate the change in output Δy and the new output y at $x = 896$ using derivatives.

B.2 Suppose that the total cost of manufacturing x units of a certain commodity is $C(x) = 2x^2 + 6x + 12$. Use differentials to approximate the cost of producing the 21st unit. Compare this estimate with the cost of actually producing the 21st unit.

B.3 A manufacturer's total cost is $C(x) = 0.1x^3 - 0.25x^2 + 300x + 100$ dollars, where x is the level of production. Estimate the effect on the local cost of an increase in the level of production from 6 to 6.1 units.

Examples

Revenues

C.1 Derive the Marginal Revenue function for the non-linear demand schedule $p = 80 - q^{0.5}$

C.2 For the total revenue function $TR = 500q - 2q^2$ find the value of the Marginal rate when $q = 80$

C.3 Find the output at which MR is zero when $p = 720 - 4q^{0.5}$ describes the demand schedule.

C.4 Derive the MR function for the demand function $q = 400 - 0.1p$.

Examples

Rates of change

D.1 A manufacturer determines that when x thousand units of a particular commodity are produced, the profit generated will be $P(x) = -400x^2 + 6800x - 12000$ dollars. At what rate is profit changing with respect to the level of production x when 9000 units are produced?

D.2 The gross domestic product (GDP) of a certain country was $N(t) = t^2 + 5t + 106$ billion dollars t years after 1998.

- D.2.1 At what rate was the GDP changing with respect to time in 2008?
- D.2.2 At what percentage rate was the GDP changing with respect to time in 2008?

Examples

Chain Rule

E.1 An environmental study of a certain suburban community suggests that the average daily level of carbon monoxide in the air will be $c(p) = \sqrt{0.5p^2 + 17}$ parts per million when the population is p thousand. It is estimated that t years from now, the population of the community will be $p(t) = 3.1 + 0.1t^2$ thousand. At what rate will the carbon monoxide level be changing with respect to time 3 years from now?

Examples

Logarithmic and exponential differentiation

F.1 A manufacturer determines that x units of a particular luxury item will be sold when the price is $p(x) = 112 - x \ln x^3$ hundred dollars per unit.

F.1.1 Find the revenue and marginal revenue functions.

F.1.2 Use marginal analysis to estimate the revenue obtained from producing the fifth unit. What is the actual revenue obtained from producing the fifth unit?